(All rights reserved.)

ADVANCE PROOF—*(Subject to revision).*

This Proof is sent to you for discussion only, and on the express understanding that it is not to be used for any other purpose whatsoever.—*(See Sec. 39 of the Constitution.)*

Canadian Society of Civil Engineers.
INCORPORATED 1887.

TRANSACTIONS.

N.B.—This Society, as a body, does not hold itself responsible for the facts and opinions stated in any of its publications.

BRIDGE CALCULATIONS.

By H. E. Vanvelet, M.Can.Soc.C.E.

To be read 10th October.

The opinion of many well known authorities is that it would be preferable to use a distributed load, that would be safe for all existing types of locomotives in use on railways, and that would leave a margin for the probable increase of weight in the future. The wheel base of a locomotive as well as the weight on each axle is limited by the radii of the curves and the section of the rail; and although the weight of cars is constantly increasing, there exists a necessary relation between the engine and train weights. In general practice, the train weight is considered as distributed and the engine weight as concentrated. The author thinks that the weight of a wheel is always distributed by the rail and ties (more so in locomotives than in cars, owing to the lesser distance between the wheels), and that both weights should be considered as distributed. It will always be necessary to use two different distributed loads, and the equivalent distributed load will vary with the length of span.

Furthermore the stresses, although calculated with the greatest care, are not the actual stresses in a bridge, and frequently discrepancies, amounting to several thousand pounds, are shown during the erection. The general practice is to have the posts fastened by pins or rivets, allowing them to work as tension members, and the top chord is rigid, instead of having articulations at every panel point. It follows that the strains are not what they would be in an articulated system, where the posts could only take compression, the differences being more especially apparent in bridges with inclined top chords, which act partially as an arc, the posts acting as suspenders. Another cause of error is the use of stringers, rivetted to the floor beams, which act as parts of the top or bottom chords, as the case may be.

It must not be supposed that the author is advocating free articulations with bolted stringers and slotted holes, as he believes that the actual practice increases the solidity of the bridges, and prefers to have stiffness in his work, at the cost of some uncertainty in his calculations. He would also follow in the lead of an eminent bridge engineer in the United States, whose trusses are rather light, a large quantity of material being used in the stiffening of the bridges, in the top and bottom laterals, sway bracing, and especially in the portals, the last of which is certainly a very important part of a bridge. Most of the actual specifications seem to be made for the perusal of outsiders more than for actual use, and it seems (to give one instance) that the rivetting foreman and inspector should know without being told, what the appearance of a rivet must be after it is driven.

In treatises on bridges, written by French authors, it is always said that we must not rely too much on calculations, and the best that can be said about their rules is that bridges built according to them and with a large factor of safety have withstood the test of time.

Experience would seem to show that, usually, the longer the specifi-

If instead of two weights P and p a weight R is distributed

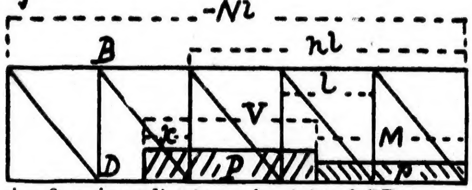
$$\therefore y = \frac{R}{2}(aL - a^2)$$

The weights P and p can then be replaced by a weight R such that

$$R = \left(1 - \frac{V}{L}\right)^2 (p - P) + P$$

$$\text{or } R = P - (P - p)\left(1 - \frac{V}{L}\right)^2$$

Fig 2



The shearing force immediately on the right of BD

$$= \frac{VP(x + nl - V_2)}{Nl} + \frac{M^2P}{2Nl} - \frac{Px^2}{2l}$$

$$= -x^2\frac{NP - p}{2Nl} + x\frac{VP + (nl - V)p}{Nl} + \frac{VP(nl - \frac{V}{2})}{Nl} + \frac{(nl - V)^2}{2Nl}p$$

Since $M = x + nl - V$

If the shearing force is to be a maximum

$$\therefore x = \frac{V(P - p) + nlp}{NP - p} \quad \text{or} \quad \frac{Px}{l} = \frac{VP + Mp}{Nl}$$

with the condition

$$V < n + nl$$

$$i.e., \; V < \frac{NPnl + V(P - p)}{NP - p}$$

$$\text{or } V < \frac{N.nl}{N - 1}$$

Hence max. shearing force

$$= \frac{V(P - p)[2NPnl - VP(N - 1)] + Nn^2l^2Pp}{2Nl(NP - p)}$$

For a uniformly distributed load the max. shearing force $= \frac{n^2Rl}{2(N-1)}$

Hence, that these two shearing forces may be equal,

$$R = \frac{(N - 1)P}{NP - p}\left\{P - (P - p)\left(1 - \frac{V}{nl}\right)^2 + \frac{(P - p)V^2}{Nn^2l^2}\right\}$$

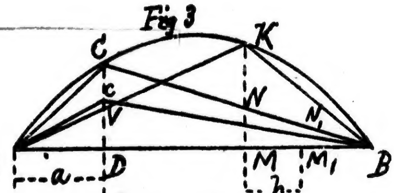
By taking

$$R = P - (P - p)\left(1 - \frac{V}{nl}\right)^2$$

we have a close approximation on the safe side. The weights P and p can be replaced without material error by one weight R; V being the length occupied by the distributed load P, and n the number of panels which must be fully loaded to give the maximum stress.

2° Graphic calculation of bending moments, taking every wheel into account.

Fig 3



If a weight P is moving along AB, the bending moment at the point of application will be:

$$y = \frac{P(l - x)}{l}x$$

can be replaced without material error by one weight R; V being the length occupied by the distributed load P, and n the number of panels which must be fully loaded to give the maximum stress.

To find the moments at D_o draw a series of triangles ABC, $A_1B_1C_1$, $A_2B_2C_2$ etc., so that

$AD = A_1D_1 = A_2D_2 = A_oD_o$

$AB = A_1B_1 = A_2B_2$ = length of span l

AA_1, A_1A_2,...being the distances between the weights.

As $AA_1A_2 = BB_1B_2 = DD_1D_2$ it will be convenient to have these distances on a scale, and the whole series of triangles may be drawn very quickly for every point at which the bending moment is required, and the sum $MN + MN_1 + MN_2$ will represent the moment at D_o when the weight P_2 is at M.

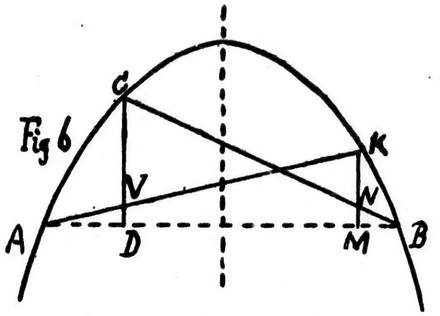
Now consider a motion of the weights betw en two consecutive apices. Each abscissa increases or decreases in proportion to the distances moved, and it is necessary to reach an apex so that one of the increasing abscissæ may decrease. A maximum can then only be reached at an apex, *i. e.*, when one of the weights is applied at D_o. It will then only be necessary to consider the ordinates at the different apices, and a curve B_2 b_1 b d_2 m a_2 a_1 d a_1 A may be drawn whose ordinates will represent the bending moments at D from the time P_2 enters the bridge until the P leaves it.

This solution brings forth a property of the parabola that the writer has never seen mentioned before, and whose limits can be enlarged by analytical demonstration.

Draw the triangle AKB (fig. 6)

$$KM = \frac{R(l - x)}{l}x \text{ and } VD = \frac{P(l - x)}{l}a = MN$$

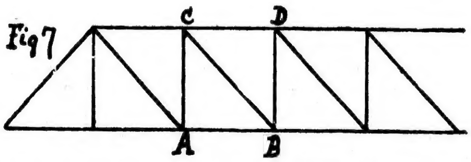
Fig 6



Join (fig. 6) two points C and K of a parabola to the points of intersection B and A, and of a perpendicular to the axis. The lengths VD and MN are equal.

3° Graphic calculation of stresses in the members of a truss, taking every wheel into account. This calculation is based on the two following theorems, for which the writer is indebted to Mr. Joseph Meyer, of the Union Bridge Co.

Fig 7



To have the maximum stress in diagonal CB (fig. 7), the sum of the weights on the left of B including the weight at B must be larger than the sum of all the weights on the truss divided by the number of panels.

To have the maximum stress in AB (fig. 7) the sum of the weights at the left of A, not including the weight at A, divided by the number of panels at the left, must be less than the sum of the weights on the bridge divided by the number of panels in the truss.

Fig 8

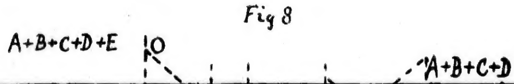
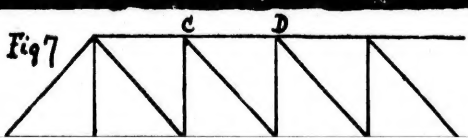
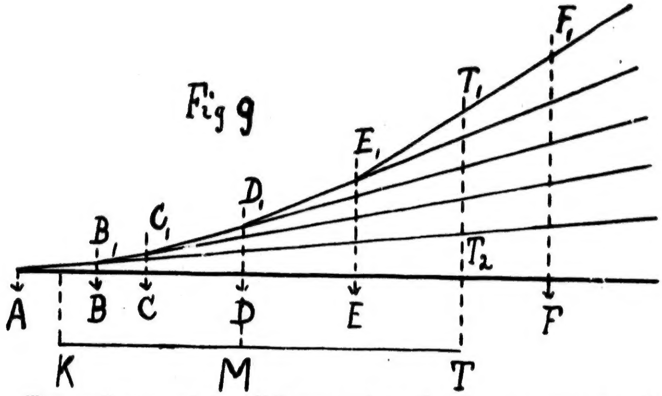


Fig 7



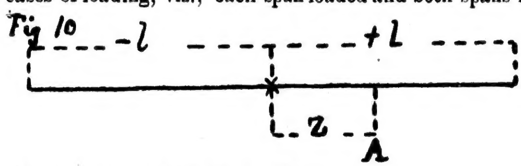


The ordinates of the different points B_1 C_1 etc., measure the moments in relation to B, C, etc., of all the weights at the left, and if we want to find the values $R \times N\,l$ and M_1 when M is at D and the ends of the truss at K and T_1, M_1 will be measured by $D_1\,D_2$ and $R \times N\,l$ by $T_1\,T_2$

Remarking that for a distributed load the polygonal curve $AB_1\,C_1\,D_1$ becomes a parabola, we could find the demonstration of many interesting properties of the parabola.

4° Bending moments in continuous bridges of 2 spans.

The maximum bending moments at each point are given by considering three cases of loading, viz., each span loaded and both spans loaded.



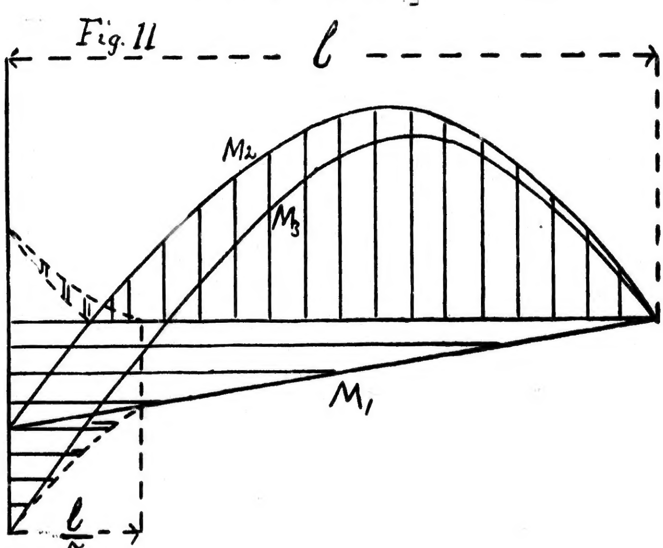
The maxima are then given by the line

$$M_1 = \tfrac{1}{16}\,pl(l-z) \qquad (1)$$

and the two parabolæ

$$M_2 = \frac{Z}{16}\,pl\,(l-z) - p\frac{(l-z)^2}{2} \qquad (2)$$

$$M_3 = \tfrac{3}{8}\,pl(l-z) - p\frac{(l-z)^2}{2} \qquad (3)$$



If a complete discussion were made, it would be found that for a length $\frac{l}{5}$ from the centre, an hyperbola intervenes, increasing the negative moments and giving also positive moments as shown by double

tions with bolted stringers and slotted holes, as he believes that the actual practice increases the solidity of the bridges, and prefers to have stiffness in his work, at the cost of some uncertainty in his calculations. He would also follow in the lead of an eminent bridge engineer in the United States, whose trusses are rather light, a large quantity of material being used in the stiffening of the bridges, in the top and bottom laterals, sway bracing, and especially in the portals, the last of which is certainly a very important part of a bridge. Most of the actual specifications seem to be made for the perusal of outsiders more than for actual use, and it seems (to give one instance) that the rivetting foreman and inspector should know without being [illegible], what the appearance of a rivet must be after it is driven.

In treatises on bridges, written by French authors, it is always said that we must not rely too much on calculations, and the best that can be said about their rules is that bridges built according to them and with a large factor of safety have withstood the test of time.

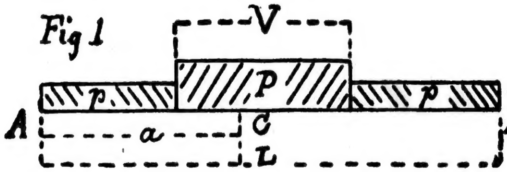
Experience would seem to show that, usually, the longer the specifications the worse the bridges are; and what may be considered to be a standard in bridges in the United States is built with a two page specification.

This paper brings forth solutions of the following problems, which the author believes to be new :—

To find the maximum bending moments and shearing stresses in girders or trusses: 1° taking into account a distributed engine load, followed and preceded by a distributed train load. 2° Taking into account the load on every wheel. A simplification in the calculation of bending moments, in continuous bridges of two spans, will also be referred to.

1° Calculation of the maximum bending moment with a distributed load P, occupying a length V, preceded and followed by a distributed load p.

Fig 1



Let y be the bending moment at c

$$y = -x^2\left(\frac{P-p}{2}\right) + x(P-p)\left(1-\frac{V}{L}\right)a - (P-p)\left[\frac{V^2 a}{2L} - Va\right]$$

$$-\frac{Pa^2}{2} + \frac{pLa}{2}$$

$$\frac{dy}{dx} = -x(P-p) + (P-p)\left(1-\frac{V}{L}\right)a$$

If y is to be a maximum $\frac{dy}{dx} = 0$ and $x = a\left(1-\frac{V}{L}\right)$*

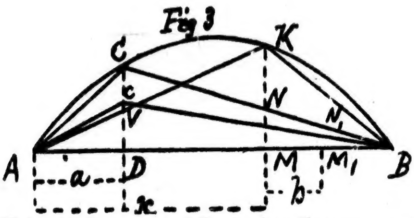
Hence, $y = -a^2\left[\left(1-\frac{V}{L}\right)^2\left(\frac{p-P}{2}\right) + \frac{P}{2}\right]$

$$-a\left[\frac{PV^2}{2L} - PV - \frac{PL}{2} + pV - \frac{pV^2}{2L}\right]$$

or $y = \left[\left(1-\frac{V}{L}\right)^2\left(\frac{p-P}{2}\right) + \frac{P}{2}\right]\left[aL - a^2\right]$

can be replaced without material error by one weight R; V being the length occupied by the distributed load P, and n the number of panels which must be fully loaded to give the maximum stress.

2° Graphic calculation of bending moments, taking every wheel into account.



If a weight P is moving along AB, the bending moment at the point of application will be:

$$y=\frac{P(l-x)}{l}x$$

Having drawn this parabola, the bending moment at a point D at a distance a from A will be:

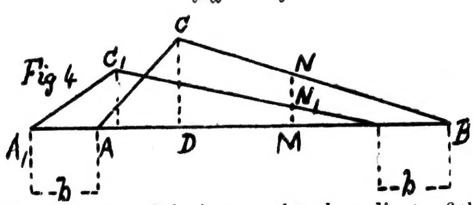
$$y=\frac{P(l-x)}{l}a$$

and CD being an ordinate of the parabola for an abscissa $x=a$

$$CD=\frac{P(l-a)}{l}a$$

In the triangle ACB we have:

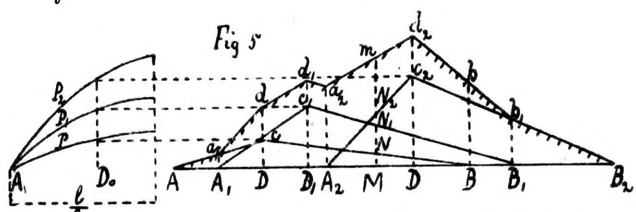
$$MN=CD\frac{l-x}{l-a}=P\frac{l-x}{l}a=y$$



The bending moment at D is then equal to the ordinate of the triangle ACB at the point of application of the weight P. For another load P_1 we should have another triangle AC_1 B_1 and if b is the distance between the two weights, $MN+M_1N_1$ will represent the bending moment at D produced by the weights P and P_1. By sliding the triangle AC_1 B a distance b to the left, M_1 comes to M, and the bending moment at D for any position of the two weights is given by the ordinates $MN+MN_1$ of the two triangles.

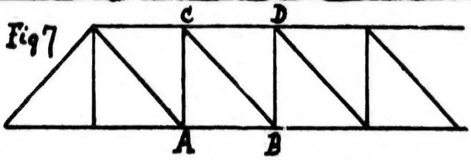
The same reasoning will apply to any number of weights.

To apply the method it is sufficient to draw the $\frac{1}{2}$ parabola $y=P\frac{(l-x)}{l}x$ to a convenient scale.



• This formula has been used, if the writer is not mistaken, by the Keystone Bridge Co.

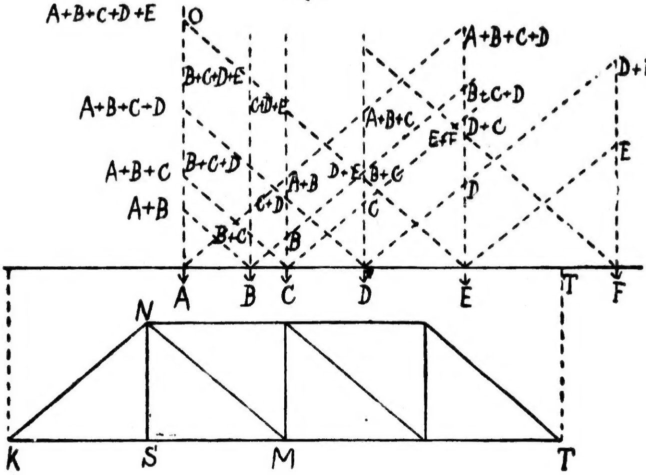
Fig 7



To have the maximum stress in diagonal CB (fig. 7), the sum of the weights on the left of B including the weight at B must be larger than the sum of all the weights on the truss divided by the number of panels.

To have the maximum stress in AB (fig. 7) the sum of the weights at the left of A, not including the weight at A, divided by the number of panels at the left, must be less than the sum of the weights on the bridge divided by the number of panels in the truss.

Fig 8



First draw the diagram shown in fig. 8, and let it be required to find the sum of weights on truss K T, and sum of weights at the left of M. The first sum be given at O by following the diagonal E O, and the second sum at V by following the diagonal A V.- By moving the truss so that M occupies the different positions A B C, etc., it will be easy to find the worst situation of the load, applying the theorems given before,

Now let M_2 be the bending moment at M

R the reaction at K

M_1 the moment in relation to M of the weights on the left of M.

Let l be the panel length

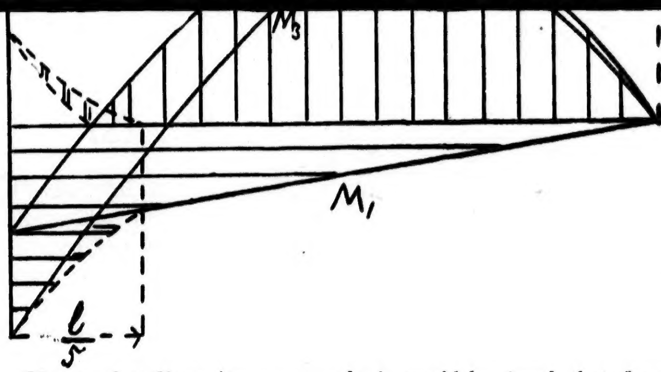
N " number of panels in truss

n " " " left of M

$$M_2 = R \times n\,l - M_1$$

$$= (R \times N\,l)\frac{n}{N} - M_1$$

$$MN = R - \frac{M_1}{l} = \frac{\frac{(R \times N\,l)}{N} - M_1}{l}$$ as, to have the maximum of M N a part only of M S is usually loaded; we want then only to know the quantities R × N l and M. To find those values we will draw another diagram.

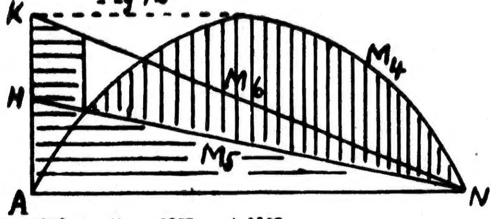


If a complete discussion were made, it would be found that for a length $\frac{l}{5}$ from the centre, an hyperbola intervenes, increasing the negative moments, and giving also positive moments as shown by double lines. But the results would not be changed materially.

If M_4 is the bending moment produced by a distributed load p on a single span l we have a parabola

$$M_4 = -\frac{pl(l-z)}{2} - \frac{p(l-z)^2}{2}$$

Fig 12



If AH = HK the lines NH and NK are

$$M_5 = \frac{pl}{16}(l-z) \text{ and } M_6' = \frac{pl}{8}(l-z)$$

and it is easy to see that

$$M_2 = M_4 - M_5$$
$$M_1 = -M_5$$
$$M_3 = -(M_6 - M_4)$$

which gives an easy method to have the bending moments.

The author thinks that the first formula given may have some practical value, and he would like to have the opinion of bridge engineers about it, as well as the opinion of mechanical engineers as to the value to be given to the constants.

The coefficients to be determined are Pp and V. In the Canadian Pacific Ry. specification p is taken at 3000, and in calculations of many bridges the writer has taken P = 3730 and V = 105′0″ and has found very little difference when taking every wheel into account.

For spans under 105 feet and over 21 feet he has taken

$$P = 4600,\ p = 3240,\ V = 21'\text{-}0''$$

and the formula becomes

$$R = 3730 - 730\left(1 - \frac{105}{nl}\right)^2 \text{ for spans over } 21'$$

$$R = 4600 - 1360\left(1 - \frac{21}{nl}\right)^2 \text{ for spans over } 21'$$

$$R = 4600 \qquad \text{for spans under } 21'$$